In Favour of Posting

香港中文大學
The Chinese University of Hong Kong

Institute of Theoretical Computer Science and Communications

ITCSC-CSE Joint Seminar

The Paulsen Problem, Continuous Operator Scaling, and Smoothed Analysis

By

Dr. Tsz Chiu KWOK

Postdoc researcher, University of Waterloo

12 January 2018, Friday

11:00 am – 12:00 nn

Room 1009, 10/F, William MW Mong Engineering Building, CUHK

Abstract:

The Paulsen problem is a basic open problem in operator theory: Given vectors $u_1,\dots,u_n \in R^d$ that are ϵ-nearly satisfying the Parseval's condition and the equal norm condition, is it close to a set of vectors $v_1,\dots,v_n \in R^d$ that exactly satisfy the Parseval's condition and the equal norm condition? Given $u_1,\dots,u_n$, the squared distance (to the set of exact solutions) is defined as $\inf_v \sum_{i=1}^{n} \|u_i - v_i\|22$ where the infimum is over the set of exact solutions. Previous results show that the squared distance of any ϵ-nearly solution is at most $O(poly(d,n,\epsilon))$ and there are ϵ-nearly solutions with squared distance at least $\Omega(d\epsilon)$. The fundamental open question is whether the squared distance can be independent of the number of vectors n.

We answer this question affirmatively by proving that the squared distance of any ϵ-nearly solution is $O(d^{13/2}\epsilon)$. Our approach is based on a continuous version of the operator scaling algorithm and consists of two parts. First, we define a dynamical system based on operator scaling and use it to prove that the squared distance of any ϵ-nearly solution is $O(d^2 n\epsilon)$. Then, we show that by randomly perturbing the input vectors, the dynamical system will converge faster and the squared distance of an ϵ-nearly solution is $O(d^{5/2}\epsilon)$when n is large enough and ϵ is small enough. To analyze the convergence of the dynamical system, we develop some new techniques in lower bounding the operator capacity, a concept introduced by Gurvits to analyze the operator scaling algorithm.

Biography:

Tsz Chiu KWOK got his BSc in Math and PhD in CS at the Chinese University of Hong Kong. He is now a postdoc researcher at University of Waterloo.

******* ALL ARE WELCOME *******

Hosted & Enquiries : Prof Siu On Chan/ Institute of Theoretical Computer Science and Communications Tel: 3943 4263/3943 3452